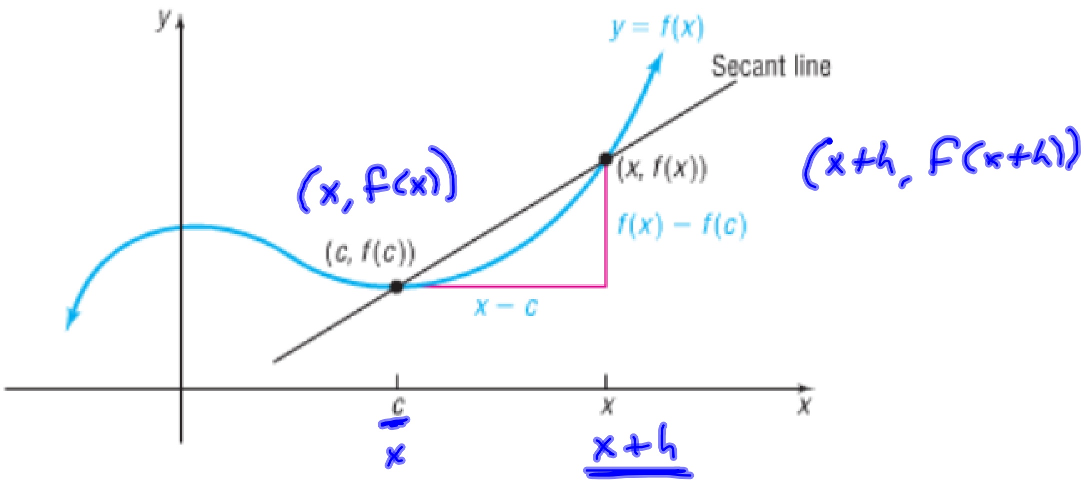
The average rate of change of a function has an important geometric interpretation. Look at the graph of $y = f(x)$ in Figure 25. We have labeled two points on the graph: $(c, f(c))$ and $(x, f(x))$. The line containing these two points is called the **secant line**; its slope is

$$m_{\text{sec}} = \frac{f(x) - f(c)}{x - c} = \frac{f(x+h) - f(x)}{h}$$

Figure 25



Slope of the Secant Line

The average rate of change of a function equals the slope of the secant line containing two points on its graph.

If c is in the domain of a function $y = f(x)$, the **average rate of change of f** from c to x is defined as

$$\boxed{\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c} \qquad x \neq c \qquad \textbf{(1)}}$$

In calculus, this expression is called the **difference quotient** of f at c.

Slope of the secant line, average rate of change, difference quotient - This is *all the same thing !!!*

56. Find the average rate of change of $h(x) = x^2 - 2x + 3$

(a) From -1 to 1 $\rightsquigarrow$ $x = 1,\ c = -1$

(b) From 0 to 2

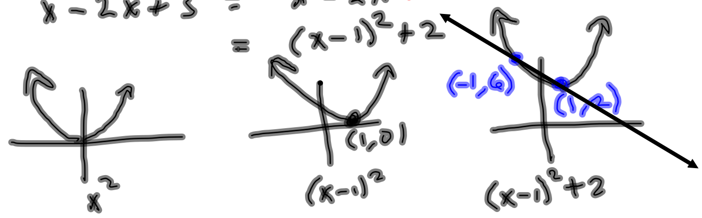
(c) From 2 to 5

(a) $\dfrac{h(1)-h(-1)}{1-(-1)} = \dfrac{(1)^2-2(1)+3-((-1)^2-2(-1)+3)}{2}$

$= \dfrac{1-2+3-(1+2+3)}{2} = \dfrac{2-6}{2} = \dfrac{-4}{2} = -2$

$x^2-2x+3 = x^2-2x+1^2-1^2+3$

$= (x-1)^2+2$



Find the average rate of change of h from x to c, i.e., find the difference quotient for h.

$\underbrace{x}_{\text{ecstasy}}$

$\dfrac{h(x)-h(c)}{x-c}$

$= \dfrac{x^2-2x+3-(c^2-2c+3)}{x-c}$

$= \dfrac{x^2-2x+3-c^2+2c-3}{x-c} = \dfrac{x^2-2x-c^2+2c}{x-c}$

$= \dfrac{x(x-2)-c(c-2)}{x-c} =$ No help

$= \dfrac{x^2-c^2-2x+2c}{x-c} = \dfrac{(x-c)(x+c)-2(x-c)}{x-c}$

$= \dfrac{\cancel{(x-c)}(x+c-2)}{\cancel{x-c}} = x+c-2$.

$\boxed{\begin{array}{l}\text{Calculus: Let } c \to x \\ \text{Then we have } x+x-2 = 2x-2 \text{ is the exact slope at } x. \\ \text{Not Tested.}\end{array}}$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{(x+h)^2-2(x+h)+3-(x^2-2x+3)}{h}$

$= \dfrac{x^2+2xh+h^2-2x-2h+3-x^2+2x-3}{h}$

$= \dfrac{2xh+h^2-2h}{h} = \dfrac{h(2x+h-2)}{h} = 2x+h-2$

$\boxed{\begin{array}{l} h \to 0 \quad 2x-2 \\ \text{Bonus } (\text{Calculus})\end{array}}$

$\dfrac{f(x)-f(c)}{x-c}$

$\dfrac{f(x+h)-f(x)}{h}$

A function f is **even** if, for every number x in its domain, the number $-x$ is also in the domain and

$$\boxed{f(-x) = f(x)}$$

Symmetric w.r.t. y-axis.

A function f is **odd** if, for every number x in its domain, the number $-x$ is also in the domain and

$$\boxed{f(-x) = -f(x)}$$

Symmetric w.r.t. origin

THEOREM:

A function is even if and only if its graph is symmetric with respect to the y-axis. A function is odd if and only if its graph is symmetric with respect to the origin.

In Problems 21–28, the graph of a function is given. Use the graph to find:

(a) *The intercepts, if any*

(b) *The domain and range*

(c) *The intervals on which it is increasing, decreasing, or constant*

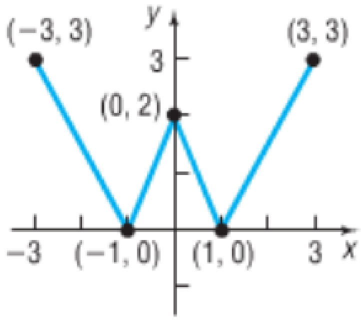
(d) *Whether it is even, odd, or neither*

22.

3

R

0



-3 D 3

$\mathcal{D} = \{x \mid f(x) \text{ is defined}\}$

$= \{x \mid -3 \le x \le 3\}$

$= [-3, 3]$

$\mathcal{R} = \{y \mid y = f(x) \text{ for some } x \in \mathcal{D}\}$

$= \{y \mid 0 \le y \le 3\}$

$= [0, 3]$

Problems 33–44, determine algebraically whether each function is even, odd, or neither.

35. $g(x) = -3x^2 - 5$

$\Rightarrow g(-x) = -3(-x)^2 - 5$

$= -3x^2 - 5 = g(x)$ $\boxed{\text{EVEN}}$

40. $f(x) = \sqrt[3]{2x^2 + 1}$

$f(-x) = \sqrt[3]{2(-x)^2 + 1}$

$= \sqrt[3]{2x^2+1} = f(x)$ is even.

$|ab| = |a||b|$

44. $F(x) = \dfrac{2x}{|x|}$

$\Rightarrow \boxed{F(-x) =} \dfrac{2(-x)}{|-x|} = \dfrac{-2x}{|x|}$

$= \dfrac{-2x}{|-1||x|} =$

$= -\dfrac{2x}{|x|}$

$= \boxed{-F(x)}$ ODD

$x^3 - 3x^5 + 9x^7$ ODD

$x^4 + 1$ EVEN

$x^3 - 3x^5 + 9$ Neither

$\dfrac{x^3 - 2x^5}{x^4 + 1}$ ODD

$\dfrac{x^5 - 2x^7}{2x^4 + 5x^3}$ EVEN

64. Constructing an Open Box An open box with a square base is required to have a volume of 10 cubic feet.

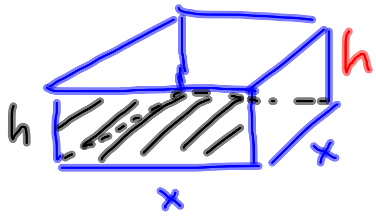
(a) Express the amount A of material used to make such a box as a function of the length x of a side of the square base.

(b) How much material is required for a base 1 foot by 1 foot?

(c) How much material is required for a base 2 feet by 2 feet?

(d) Graph $A = A(x)$. For what value of x is A smallest?

Similar problem situation to #63, which I'm assigning for homework. But for #64, they're giving you slightly different info and are looking for slightly different things...



$V = x^2 h = 10$

$A = x^2 + 4xh$

$h = \frac{10}{x^2}$

$x^2 + 4x \cdot \frac{10}{x^2} = \boxed{x^2 + \frac{40}{x} = A(x)}$

Area as func. of x.

(b) $A(1) = 1^2 + \frac{40}{1} = 41$ square feet.

NOT AREA TIMES X.

64. a. Let A = amount of material,
x = length of the base, h = height, and
V = volume.

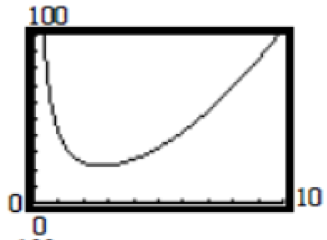
$$V = x^2 h = 10 \Rightarrow h = \frac{10}{x^2}$$

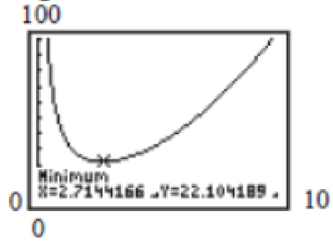
$$\text{Total Area } A = (\text{Area}_{\text{base}}) + (4)(\text{Area}_{\text{side}})$$
$$= x^2 + 4xh$$
$$= x^2 + 4x\left(\frac{10}{x^2}\right)$$
$$= x^2 + \frac{40}{x}$$
$$A(x) = x^2 + \frac{40}{x}$$

b. $A(1) = 1^2 + \frac{40}{1} = 1 + 40 = 41 \text{ ft}^2$

c. $A(2) = 2^2 + \frac{40}{2} = 4 + 20 = 24 \text{ ft}^2$

d. $y_1 = x^2 + \frac{40}{x}$





The amount of material is least when $x = 2.71$ ft.